Noetherian rings with small profiles

B. Saraç

Hacettepe University, Ankara, Turkey

a joint work with

S.R. López-Permouth[1]

[1]*Ohio University, Athens, Ohio, United States*

Abstract

It is a well-known fact from torsion theory that the collection of all hereditary pretorsion classes in a module category is in one-to-one correspondence with a set and that it can be given a lattice structure. Given any ring R, the sublattice of hereditary pretorsion classes of right R-modules containing all semisimple modules is called the (right) profile of R. It is clear that a ring R is semisimple Artinian if and only if its profile is a singleton. In this talk, we present several interesting properties and characterizations, under some circumstances, of right Noetherian rings whose profiles contain only two or three elements. We also give some particular examples of such rings. Mojority of the results to be presented are from [9] and [10].

Keywords

Noetherian rings, hereditary pretorsion classes, relative injectivity, quasi-injective modules, QI rings.

References

[1] A. N. Alahmadi, M. Alkan, S. R. López-Permouth, *Poor modules: the opposite of injectivity*, Glasg. Math. J. **52**, no. A (2010) 7–17.

[2] A. K. Boyle, *Hereditary QI-rings*, Trans. Amer. Math. Soc. **192** (1974) 115–120.

[3] J. H. Cozzens, *Homological properties of the ring of differential polynomials*, Bull. Amer. Math. Soc. **76** (1970) 75–79.

[4] N. Er, *Rings characterized via a class of left exact preradicals*, Proc. Edinburg Math. Soc. **59**, No. 3 (2016) 641–653.

[5] N. Er, S. R. López-Permouth, N. Sökmez, *Rings whose modules have maximal or minimal Injectivity domains*, J. Algebra **330**, No. 1 (2011) 404–417.

[6] K. R. Goodearl, *Singular Torsion and the Splitting Properties*, Memoirs of the American Mathematical Society, **124** (American Mathematical Society, Providence,1972)

[7] L. S. Levy, J. C. Robson, *Hereditary Noetherian prime rings and idealizers*, Mathematical Surveys and Monographs, **174** (American Mathematical Society, Providence, RI, 2011).

[8] S. López-Permouth and J. E. Simental, *Characterizing rings in terms of the extent of the injectivity and projectivity of their modules*, J. Algebra, **362** (2012) 56–69.

[9] B. Saraç, *On rings whose quasi-injective modules are injective or semisimple*, submitted.

[10] B. Saraç, S. R. López-Permouth, S. Zamora-Erazo, *Rings without a middle class from a lattice-theoretic perspective*, Mediterr. J. Math. **17**, No. 2, Art. 55 (2020) 27 pp.